Uplink and Downlink Channel Capacity of Massive MIMO Enabled UAV Communications Links

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Abstract
Enhanced communication support to UAV networks can be achieved by integrating UAVs into existing cellular networks as aerial users. But there are many challenges and inadequacies in integrating UAVs with cellular networks. By utilizing multiple antennas at ground base stations, these inadequacies of existing cellular networks can be mitigated. With the use of massive MIMO technology, wherein cellular base stations are mounted with hundreds of antennas, the performance of UAV communication links gets significantly improved. In this article, we present performance evaluation of massive MIMO enabled UAV communication links, by initially covering UAV cellular communications along with its potential benefits and challenges. We then carry out the performance evaluation of UAV communication links by using basic multiple antenna techniques, shortcomings of using point to point MIMO and Multi user MIMO (MU-MIMO). Lastly we derive uplink and downlink channel capacity expressions for evaluating massive MIMO enabled UAV communication links along with few numerical results.

Key-words: Multiple Input Multiple Output (MIMO), Multi User MIMO (MU-MIMO), 5G Communication, Long Term Evolution (LTE).

1. Introduction
An unmanned aircraft, which is piloted by a remote control unit or by an on board computer is called an Unmanned Aerial Vehicle (UAV). Traditionally UAVs have been used for military
applications of battlefield & airspace surveillance and patrolling. However, with the advancements in technology particularly in field of miniaturization of electronic instruments and improvements in control systems, civil applications of UAVs are growing. Civil applications include search operations during natural disasters, crowd or traffic surveillance, wildlife conservation, goods transportation etc [1]-[3]. Easily available low cost quadcopters drones are being regularly used for drone racing and aerial photography. Many such civil applications will emerge in future [4]–[6]. UAVs can also be put into use as aerial platforms for communication such as airborne base stations (BSs) or flying relays. Such arrangement is commonly called UAV assisted communication, which is realized by mounting the communication transceivers to UAVs, for provisioning or enhancing communication services to the users on ground [7], [8]. Similarly, cellular connected UAVs are the one which are used as aerial users/nodes [9], [10]. A swarm of UAVs creating Flying Ad Hoc Networks (FANETs) are being used to support high rate wireless communications in a large geographical area [11].

1.1 Cellular Communication Support to UAVs

Communication link between UAV and its ground station is generally established by use of unlicensed spectrum. The range between a ground station and UAV is primarily limited by unlicensed frequency spectrum used and transmit power. Enhanced ranges can be achieved by already deployed cellular networks, which are no more limited by unlicensed frequency spectrum. If the cellular network delivers the data rate required by a UAV, then UAV will have same range as the coverage of cellular network. Figure 1 shows the connectivity of UAV with ground station using one to one link. Figure 2 shows the connectivity of UAV with a cellular network. Cellular connectivity can support communication between UAVs, between UAV and its operator and between UAV & traffic control. Thus, already established cellular communication networks are optimum choice to support UAV communications. Cellular networks have the resources and potential to fulfill all requirements of UAV communication in terms of global availability, dedicated frequency spectrum, secure channels, SIM (Subscriber Identity Module) and IMEI (International Mobile Equipment Identity) based Identification and superior efficiency.
1.2 Existing Studies and Tutorials

Few surveys related to UAV communications have been published in recent past [11]-[14], bringing out the characteristics, requirements and various issues in UAV communication systems. Particularly the communication perspectives for civil applications of UAVs were reviewed in [11]. The issues encountered in provisioning of reliable and stable wireless UAV communication were discussed in [14]. In [15], authors described potentials of provisioning of IoT services from sky based low altitude UAVs communications. Simulated and actual cyber attacks were discussed for reviewing cyber security of UAVs in [16]. In [17], author compared various routing protocols for UAVs, for networking. Propagation channel modeling for air to ground channels was surveyed in [18]. Various measurement methods for channel modeling for UAVs along with characteristics were discussed in [19]. In order to improve UAV mission time, the authors presented various wireless charging techniques in [20]. Protocols and methods of airborne communication network designs were discussed in [21].

1.3 Paper Contributions and Organization

Although many viewpoints of performance evaluation of UAV communication links have been provided through various existing studies, still there is a need to discuss fresh perspective of performance metrics, by utilization of multiple antennas in base stations. In this paper, we are aiming to give the reader basic understanding of UAV cellular communications. The rest of this paper is outlined as follows: In Section 2, we envision an overview of potential benefits and challenges of provisioning of cellular communication for UAVs. In section 3, we discuss performance of various
basic multiple antenna techniques that can be used for providing cellular communication support to UAVs. In Sections 4 & 5, we explain Multi user MIMO enabled UAV communications and massive MIMO enabled UAV communication respectively. In Section 6, we bring out certain numerical results and key insights. Lastly in Section 7, we present our conclusion.

2. Challenges and Inadequacies in Implementation of Cellular Communications for UAVs

UAVs need features such as high mobility, long range, low latency and high throughput for their communications, which could only be provided by a fresh advance technology [22]. Advance cellular communication technologies when used for UAV communications, provide numerous benefits. Apart from regular telemetry information, a cellular connected UAV gets real time air traffic information, emergency incidents information and weather information. Cellular connectivity enables remote operation of all prior flight manual tasks of UAV operator. Currently deployed, 4G Long Term Evolution (LTE) has wide range of capabilities to support UAV communication. Next generation of cellular technology, 5G has enhanced capabilities to connect more devices at higher data rates. Therefore, 4G/5G cellular communication technologies can be effectively utilized for UAV communications. However, implementing cellular communications for UAV, is a challenge in terms of deployment, channel modeling, network security, energy efficiency etc [23], [24]. The implementation of cellular communication support to UAVs has following challenges:

- **Channel Models**: Being aerial objects, UAVs have typical channel characteristics of 3D space and time variations. These characteristics cause increased complexity of air to ground channels as compared to ground to ground channels. Therefore, for characterizing air to ground channels, conventional channel models are not suitable. UAV to ground channels are dependent on elevation angle, operational altitude, propagation environment etc. Accurate channel models are essential for evaluating system performance.

- **Mobility and Deployment**: UAVs are aerial objects having high mobility and specific channel characteristics. To reduce physical collisions and handovers, these aspects are required to be considered for optimal deployment.

- **Trajectory Control**: Each UAV follows a particular trajectory in air based network comprising of set of UAVs. UAVs have to establish simultaneous links with neighboring UAVs as well as ground user. As a result, because of practical constraint, the identification of optimal flying trajectory for UAVs is a complex task. Therefore, trajectory control is essential to enhance link probability and maintain full coverage of complete target area.
• **Altitude of Operation:** Because of size, weight and power constraints of UAVs, different variants of UAVs have different altitudes of operation. Lower altitude of UAV reduces path loss, whereas higher altitude increases LOS connectivity. Therefore, a trade off is required to be struck between path loss and LOS connectivity, by selection of different altitudes for UAVs.

• **Management of Interference:** Co-channel interference is experienced between air to ground channels and ground cellular network. Similar interference may also be experienced between different air to air channels. This may lead to major disruptions in air interfaces. Therefore, appropriate management of interference is a challenge in UAV communications.

• **Energy Limitation:** The mission operating time of UAV is limited by power constraints or energy consumption, which is provided by battery of UAV. Therefore, advance charging technologies are required for longer and persistent operation of UAV mission.

• **Backhaul Links:** Large bandwidth wired links are used for backhaul between ground base stations and core networks. High capacity wireless links are used for backhaul between UAV base station and ground base station. QoS of both aerial and ground users will be limited by these backhaul links.

• **Security of Network:** Cellular networks providing simultaneous communication to aerial and ground users are vulnerable to malicious attacks. This is because of distinct characteristics of both type of users and broadcasting features of LOS transmissions. Therefore, it is essential to formulate network security measures to safeguard cellular networks.

The existing cellular communication networks are found to be non appropriate for few typical requirements of UAVs communications. This is primarily because of the fact that antennas of ground base station are generally tilted towards ground, thereby providing cellular coverage at lower elevation angles and lower altitudes only [25]- [28]. Other major reasons are channel interference from neighboring cells and distinct aerial mobility patterns of UAVs, The inadequacies of existing cellular networks can be removed by utilization of multiple antennas in base stations, because use of multiple antennas have inherent benefits in terms of Beamforming gain, Spatial multiplexing and Spatial diversity.

3. **Performance of Basic Multiple Antenna Techniques for UAV Communication**

It is understood that the inadequacies of existing cellular networks in providing optimum communication support to UAVs, can be removed by utilization of multiple antennas in base stations.
We now carry out performance analysis of various multiple antenna techniques, that can be suitably employed for cellular communication support to UAVs.

3.1 Measure of Communication Performance

For a basic communication channel as shown below, $x$ is transmitted symbol with $q$ power, $\beta$ is channel gain, $n$ is noise and $y$ is received signal. All values are complex valued numbers. The receiver guesses the value of $x$ based on received signal $y$.

$$y = \sqrt{\beta} x + n$$

The channel capacity $C$ is given as, $C = \log_2 (1 + \frac{q\beta}{N_o})$ bits per symbol, where, $q$ is energy per symbol, $\beta$ is channel gain, $N_o$ is noise variance. Channel capacity is number of bits per symbol that can be transmitted without error. Therefore, it is the performance metric for analyzing communication systems. As the signal is complex valued, it is represented by $B$ complex samples per second where, $B$ is bandwidth. Therefore, in the capacity expression number of symbols is $B$ symbols per second. Channel capacity expression becomes $C = B \log_2 (1 + \frac{q\beta}{N_o})$ bits per second. Here, $q = \frac{P}{B}$, $P$ is power and $B$ is number of symbols per second. Therefore, channel capacity expression becomes $C = B \log_2 (1 + \frac{P\beta}{BN_o})$ bits per second, where, $\frac{P\beta}{BN_o}$ is signal to noise ratio (SNR).

3.2 Basic Multiple Antenna Techniques for UAV Communication

Single input single output (SISO), single input multiple output (SIMO), multiple input single output (MISO) and multiple input multiple output (MIMO) are basic forms of multiple antenna techniques. Diagrammatical representation of these techniques is given in Figures 3 below.
For a basic communication channel, where $x$ is energy per symbol, $g$ is channel response & $\beta$ is channel gain i.e $\sqrt{\beta} = g$ or $\beta = g^2$. SISO communication set up has single antenna to transmit and single antenna to receive. The channel capacity is given as $C = \log_2(1 + \frac{q|g|^2}{N_o})$ bits per symbol. For SIMO technique, the communication set up has single antenna to transmit and M antennas to receive. The channel capacity is given by $C = \log_2(1 + \frac{q\|g\|^2}{N_o})$ bits per symbol, $\|g\|^2$ is squared norm of channel vector i.e sum of absolute values of square of channel responses for each of M antennas. $\|g\|^2 = \sum_{m=1}^{M}|g_m|^2$, if the channel responses are same, we get M times strong signal (beamforming gain) $\frac{|g_H^H y|}{\|g\|}$. For MISO, the communication setup has M transmit antenna and one receive antenna. The channel capacity is given by $C = \log_2(1 + \frac{q\|g\|^2}{N_o})$ bits per symbol. In the channel capacity of SIMO and MISO, M times larger SNR is achieved, when transmission is done with M antennas and reception with M antennas. When transmission done with M antennas, transmission happens in directive way using beamforming towards UAV i.e M copies of signals are constructively added at UAV side without using more power. When reception is done with M antennas, one antenna (isotropic) is actively transmitting, different copies of signal are being observed, with different channel responses, all copies are added constructively using Maximum ratio combining. M noise terms are not constructively combined. In both transmission and reception using multiple antenna, beamforming gain, proportional to M is achieved.
3.3 Point to Point Multiple Input Multiple Output (MIMO) Enabled UAV Communication

Consider K transmit antennas and M receive antennas. Between transmit and receive antennas, we have scalar channel responses from transmit antenna k to receive antenna m, \( g_{m,k} \). There are total of MK channel responses to be described. Therefore, it is convenient to put them into matrix, where rows describe receive antenna and columns describe transmit antenna. 

\[
G = \begin{bmatrix}
g_{1,1} & \cdots & g_{1,K} \\
\vdots & \ddots & \vdots \\
g_{M,1} & \cdots & g_{M,K}
\end{bmatrix}
\]

The received signal at \( m \)th antenna is given by 

\[
y_m = \sum_{k=1}^{K} g_{m,k} x_k + n_m,
\]

where, \( g_{m,k} \) is channel response from \( k \)th transmit antenna to \( m \)th receive antenna, \( x_k \) is signal from \( k \)th transmit antenna and \( n_m \) is noise at \( m \)th receive antenna. Here, \( y = Gx + n \) with 

\[
\begin{bmatrix}
y_1 \\
\vdots \\
y_M
\end{bmatrix} = \begin{bmatrix}
g_{1,1} & \cdots & g_{1,K} \\
\vdots & \ddots & \vdots \\
g_{M,1} & \cdots & g_{M,K}
\end{bmatrix} \begin{bmatrix}
x_1 \\
\vdots \\
x_M
\end{bmatrix} + \begin{bmatrix}
n_1 \\
\vdots \\
n_M
\end{bmatrix}
\]

Consider model given in Figure 4, to find the capacity of MIMO channel.

![Figure 4](image1)

Transmitter Processing

![Figure 5](image2)

Receiver Processing

The transmitted signal is \( x = V\tilde{x} \) and receiver processing is \( \tilde{y} = U^y \). Ideally \( \tilde{y} \) should be equal to \( \tilde{x} \). \( U \) and \( V \) are left and right singular vectors for matrix \( G \), by singular value decomposition \( G = U \Sigma V^H \). If \( S \) is the rank of the channel matrix \( G \) such that \( S \leq \min(M,K) \). The singular values \( S_1 \geq \ldots \geq S_{\min(M,K)} \geq 0 \). Then, the channel can be parallelized i.e by this the point to point MIMO is converted into \( S \) parallel SISO channels having no interference in between (Figure 5). The representation of singular vector decomposition for \( S=3 \) i.e 3 transmit antenna and 3 receive antenna is shown in Figure 6. The channel is divided into three parallel sub channels. For each one them, there is a precoding vector \( V = [v_1 \ v_2 \ v_3] \) which is selected from their matrix \( V \) (one of the columns). Combining vector \( U = [u_1 \ u_2 \ u_3] \), is at the receiver side, the strength of sub channel is given by singular value \( S_1, S_2, S_3 \). \( S_1 \) is stronger than \( S_2 \) and \( S_2 \) is stronger than \( S_3 \). Blue is direct path, green & red are scattered paths.
Singular value decomposition creates independent parallel sub channels. The capacity is known for each of the independent channels. Rate of sub channel $k$ is $C = \log_2(1 + \frac{q_k S_k^2}{N_o})$ bits per symbol, where $q_k$ is transmit power, $S_k$ is singular value and $N_o$ is noise power spectral density. Sum rate of all sub channels i.e S parallel channels is given by $\sum_{k=1}^{S} \log_2(1 + \frac{q_k S_k^2}{N_o})$. There is a need to maximize it by selecting $q_1, ..., q_S$. Take total transmit power $q$ and divide it into $S$ parallel channels. Therefore, the capacity of point to point MIMO channel is given by

$$C = \max_{q_1 \geq 0, ..., q_S \geq 0} \sum_{k=1}^{S} \log_2(1 + \frac{q_k S_k^2}{N_o})$$ bits per symbol, $\sum_{k=1}^{S} q_k = q$

As per Water Filling method of power allocation, in case of low SNR, use only one sub channel and in case of high SNR, give equal power to all sub channels. For SIMO and MISO channel capacity $C = \log_2(1 + \frac{q\|g\|^2}{N_o})$ bits per symbol, the main benefit is beamforming gain, where SNR grows proportionally with number of antennas. This means a lot for low SNR, but not for high SNR. For MIMO capacity $C = \max_{q_1 \geq 0, ..., q_S \geq 0} \sum_{k=1}^{S} \log_2(1 + \frac{q_k S_k^2}{N_o})$ bits per symbol, the main benefit is Multiplexing gain, which is much larger thing. It is summation over number of channels. If number of transmit and receive antenna are increased, the capacity is increased linearly. Therefore, after carrying out the performance analysis of various basic multiple antenna techniques for provision of cellular communication support for UAVs, it is ascertained that there is an opportunity to improve the communication performance of the UAV communication links, particularly by increasing the number of antennas.
3.4 Problems with Point to Point MIMO Enabled UAV Communication

The multiplexing gain $S$ is equal to rank ($G$) of channel matrix. Its benefit is that if $S$ is rank of channel matrix, then $S$ signals can be spatially multiplexed at the same time and $S$ times larger capacity is obtained. For LOS, $S \approx 1$ and for NLOS $S = \min (M,K)$. Mainly beamforming gain is obtained, high SNR is likely to be LOS, which gets benefitted from multiplexing gain and low SNR is likely to be NLOS, which does not get benefitted from multiplexing gain. As shown in Figure 7, there is not much difference in capacity at low SNR (Faster scaling), at high SNR high capacity (NLOS) is achieved but not much multiplexing gain. Also the multiplexing gain is given by min ($M,K$), therefore, there is a necessity to have multiple antennas and UAVs can not have many antennas.

The option to improve is to consider Multiuser MIMO (MU-MIMO), where base stations have multiple antennas and UAV or any other user has single antenna. In MU-MIMO as shown in Figure 8, the uplink is links from UAVs to base station i.e multi point to point MIMO, as different UAVs at different locations transmit different signals at the same time. The downlink is the links from base stations to UAVs i.e point to multipoint MIMO as different beams are transmitted from base station to different UAVs at same time and frequency. It is called MIMO because base station has multiple antenna and UAVs are located at multiple points where antennas are located, even if UAV/user has single antenna. Hereafter in this paper, UAVs shall be considered as any other user of cellular network.

Assume that two UAVs want to communicate with base station, power per UAV is $P$ watts, bandwidth is $B$ and noise power spectral density $N_0$. Orthogonal multiple access means UAVs need to share bandwidth in orthogonal manner i.e divide bandwidth as $\alpha B$ to UAV1 and $(1-\alpha)B$ to UAV2. If $\beta$
is the channel gain, both UAVs have same channel quality. Two achievable rates are computed as 
\[ R_1 = \alpha B \log_2 \left( 1 + \frac{p_B}{\alpha B N_0} \right) \] and 
\[ R_2 = (1 - \alpha) B \log_2 \left( 1 + \frac{p_B}{(1 - \alpha) B N_0} \right). \]
For different values of \( \alpha \), different rates are achieved. There is a need to identify perfect operating point between two rates. With non orthogonal multiple access, both UAVs transmit simultaneously at same bandwidth and rate/capacity regions helpful in achieving all operating points are obtained by means of time sharing. That is the motivation to serve multiple users at same time in uplink. Consider \( K \) single antenna UAVs and \( M \) base station antennas as shown in Figure 9.

Let \( g_i^j \) be the channel response from UAV \( i \) to antenna \( j \), where \( i \) is the UAV transmitting the signal and \( j \) is the base station antenna receiving the signal. The signals transmitted by \( K \) UAVs are called data signals \( x_1, \ldots, x_K \) and signals received at base station are called \( y_1, \ldots, y_M \). Signals from UAV 1 are received by all antennas at base station. Similarly for each UAV, mix of signals from multiple UAVs are received by all antennas at base station and its task is to separate them and that would be particularly easy if there are at least as many base station antennas as UAVs.

4. Multiuser MIMO (MU-MIMO) Enabled UAV Communication

The received signal \( y \) is given as 
\[ y = \sqrt{\rho_{ul}} G x + w, \]
where \( \sqrt{\rho_{ul}} \) is normalizing SNR i.e noise variables are also included in \( \sqrt{\rho_{ul}} \) and thus, \( w \) has \( I_M \). Writing these signals in matrix form
\[ y = \begin{bmatrix} y_1 \\ \vdots \\ y_M \end{bmatrix}, \quad G = \begin{bmatrix} g_1^1 & \cdots & g_K^1 \\ \vdots & \ddots & \vdots \\ g_1^K & \cdots & g_K^K \end{bmatrix}, \quad x = \begin{bmatrix} x_1 \\ \vdots \\ x_K \end{bmatrix}, \quad w = \begin{bmatrix} w_1 \\ \vdots \\ w_M \end{bmatrix}. \] The parameters are normalized i.e SNR is \( \rho_{ul} \), each UAV signal is power limited as \( E[|x_K|^2] \leq 1 \) and normalized noise \( w \sim \text{CN}(0,I_M) \). This is same as point to point MIMO, but with slight differences.
• UAVs do not cooperate: $x_1, \ldots, x_k$ are independent data signals, which is very different from point to point MIMO case where orthogonal vector $x$ was created by taking an information signal multiplied with the matrix coming from singular value decomposition of the channel and that way take the information and spread it out over antennas in different ways. But here, these are independent signals as UAVs have own signals to be transmitted.

• Each UAV cares about own performance: Instead of one performance matrix describing complete system, k user capacities are to be cared instead of just one capacity. Even if spatial multiplexing is done, S different signals in point to point MIMO, S parallel channels and sum of capacities needs to be cared. Here, k UAVs are interested in their specific rate of capacities.

• Each UAV has its own power budget: In point to point MIMO, where signals are spread over different sub channels and complete power is distributed among various sub channels. Here, every UAV has its own power, own power amplifier and battery.

• The channel matrix $G$ is modeled differently from point to point MIMO case: Even if the equation $y = Gx + n$ is similar. Each column of $g$ is modeled as SIMO channel i.e one UAV to multiple antennas of base station, that column can be modeled as earlier one but different columns are modeled differently because different UAVs are at different propagation conditions. Thus, there is bigger chance that channel matrix $G$ has good properties. High rank gives possibility of communication with more data.

4.1 Sum Capacity of MU-MIMO Enabled Communication Uplink (UL)

For $y = \sqrt{\rho_{ul}} Gx + w$, where $\sqrt{\rho_{ul}}$ is SNR, $G$ is channel matrix, $x$ is transmitted signal and $w$ is noise vector. Assume channel $G$ is deterministic, all UAVs are transmitting with full power. $x \sim \text{CN}(0, I_M)$, covariance matrix is identity matrix because all elements in $x$ are independent of each other and have noise variance 1. Therefore, it is like point to point MIMO channel, but with a suboptimal signal covariance matrix $Q = I_M$, which was not the optimal choice because UAVs are not cooperating. The sum rate is given as, Sum rate: $R_1 + R_2 \ldots. + R_k = \log_2 \left( \det(I_M + \rho_{ul} G G^H) \right)$. This is sum capacity of MU-MIMO enabled UAV communication system. Achieved by successive interference cancellation, decoding order determines who gets which share. As done earlier in case of non orthogonal access. For UL capacity region $k=2$, where region contains all $(R_1, R_2)$ satisfying $R_1 \leq \log_2 \left( 1 + \rho_{ul} \|g_1\|^2 \right)$ and $R_2 \leq \log_2 \left( 1 + \rho_{ul} \|g_2\|^2 \right)$. Thus, $R_1 + R_2 = \log_2 \left( \det(I_M + \rho_{ul} G G^H) \right)$ where $G = [g_1, g_2]$. It can be
concluded that large multiplexing gains are hard to achieve in practice in point to point MIMO technique. Multi user MIMO is a similar system model but has key differences in terms of independent UAVs, different power and different performance. It has capacity & rate regions and orthogonal & non orthogonal access.

5. Massive MIMO Enabled UAV Communication

5.1 MU-MIMO vs Massive MIMO Enabled UAV Communication

Conventional MU-MIMO has base station antennas $M \leq 8$, UAVs per cell $K \leq 4$ and is used in LTE, WiFi, etc, which seldom reaches to min ($M,K$) = $K$ capacity gain. In these cases multiplexing gain that is achieved in point to point system is the minimum number of transmit and receive antennas. Then ideally MU-MIMO enabled UAV communication system should provide capacity gain compared to size of system equal to minimum number of base station antennas and UAVs, which is equal to number of UAVs as there are fewer UAVs compared to base station antennas. However, these type of gains are seldom achieved here because it is hard to operate these systems considering in practice, channel estimation is taken into account. Therefore, Massive Multiuser MIMO called Massive MIMO in short, is used to deals with this problem. Massive MIMO enabled UAV communication system has base station antennas $M \approx 100$ and UAVs per cell $K \approx 10$ or more. Massive MIMO has more directive signals, less randomness in channels, larger beamforming gain and less interference between UAVs. Characterizing feature of Massive MIMO are much more antennas than UAVs and min($M,K$)=$K$ ideally achievable capacity gain.

5.2 Channel Coherence

Wireless communication channel is linear time invariant (LTE) or not? The linearity is guaranteed by the Maxwell equation. When transmitter, receiver or something in propagation is changing, then it is not time invariant. Wireless communication channel is generally not time invariant. However, when particular short duration of time is considered, then it is approximate time invariant. The time which is approximately time invariant is the coherence time ($T_C$). It is the time where everything known can be utilized to analyse this channel. Coherence time is given by $T_C = \frac{\lambda}{2v}$, where,
\( \lambda \) is wavelength and \( v \) is speed i.e how much time does it takes to move half wavelength or when the transmitter or receiver has moved half wavelength then the channel can not change substantially.

When operating within coherence time, the channel from transmitter to receiver can be analyzed as time invariant system. However, there is one more property, that is whether the channel is time dispersive or not i.e when signal is transmitted then whether this signal gets spread out in time or not. Some dispersion is natural in wireless communication because of typical multiple propagations with time delay, with distance spreading out the signal over time. However, in frequency domain, frequency is changed, there is dispersion/variations, but on assumption or selection of short duration/interval of frequency, then it appears that frequency response is constant. This is called coherence bandwidth (\( B_c \)), the bandwidth over which the frequency response \( G(f) \approx g \) is almost constant. That means on going back to time domain, the channel response is \( g(t) = g. \delta(t) \), where \( g \) is constant. Therefore, to represent this channel only complex valued constant is required to be known. Coherence bandwidth varies a lot depending on different scenarios, sometimes there are more rapid changes and sometimes less rapid changes. Therefore, coherence bandwidth is given by \( B_c = \frac{c}{|d_{\text{max}}-d_{\text{min}}|} \) Hz, where \( d_{\text{max}} \) is the distance of maximum propagation delay and \( d_{\text{min}} \) is shortest propagation path. Coherence bandwidth is inversely proportion to Path length difference.

Block fading model of coherence interval is shown at Figure 10. Divide bandwidth into different pieces such that each piece has width equal to \( B_c \) and divide time resources into column intervals equal to \( T_c \). Each block is coherence interval with bandwidth as coherence bandwidth \( B_c \) and time interval as coherence time \( T_c \). Within one coherence interval there is a constant channel but it is described as only one scalar i.e channel between one transmitter antenna and one receiver antenna. As per Nyquist channel sampling theorem, channel interval is described by a scalar \( \tau_c = B_cT_c \) complex samples within a coherence interval. How many times this channel can be used within a coherence interval can be figured out. Therefore, operation of communication system can be broken down into coherence intervals and within coherence intervals, channel behavior needs to be learned. It can be used as multi carrier system, where each of coherence interval represents one sub carrier or one set of sub carriers. This is an example of fast fading channel.
5.3 Motivation for Massive MIMO Enabled UAV Communications

- **Favourable Propagation**: For K=2, Sum capacity is given as $R_1 + R_2 = \log_2\left(\det(I_M + \rho_{ul}GG^H)\right) = \log_2\left(\det(I_M + \rho_{ul}G^HG)\right)$. If $G = [g_1, g_2]$, then $G^H G = \begin{bmatrix} \|g_1\|^2 & g_1^H g_2 \\ g_2^H g_1 & \|g_2\|^2 \end{bmatrix}$.

Expanding the sum capacity $\log_2\left(\det(I_M + \rho_{ul}G^HG)\right) = \log_2(1 + \rho_{ul} \|g_1\|^2) + \log_2(1 + \rho_{ul} \|g_2\|^2) - \rho_{ul}^2 |g_1^H g_2|^2 \leq \log_2(1 + \rho_{ul} \|g_1\|^2) + \log_2(1 + \rho_{ul} \|g_2\|^2)$. Here, first term $\log_2(1 + \rho_{ul} \|g_1\|^2)$ is the capacity of UAV 1 and the second term $\log_2(1 + \rho_{ul} \|g_2\|^2)$ is the capacity of UAV 2. The equality in above equation will be if and only if $g_1^H g_2 = 0$. If two channel vectors $g_1^H g_2$ have inner product 0, then sum capacity is equal to the capacity of individual UAVs. Capacity region is square because there is no interference at all as both UAVs are orthogonal vectors. That is what is required to be achieved in practical system. The aim is to somehow achieve that channel vectors of different UAVs to be orthogonal. That is the motivation of massive MIMO called Favourable Propagation. Consider two M antenna channels $g_1$ & $g_2$. The inner product $|g_1^H g_2|/M$ converges to zero as $M \to \infty$. That means there is less interference between UAVs when there are many antennas. Less interference is because of beamforming gain and beamwidth i.e focusing the signal towards UAV and power of focusing in not creating any power, it means more and more focused signals are sent and focusing the
power towards the receiver and then less interference is going to be leaked into other directions. As soon as UAVs are not at the same location they will see less interference.

- **Channel Hardening:** Another motivating property of massive MIMO is Channel hardening. Consider M antenna channel with channel vector $g \sim \text{CN} (0, I_M)$, covariance matrix is identity matrix because all elements of $g$ are independently distributed. Normalized channel gain $\frac{\|g\|^2}{M}$, has mean $\frac{M}{M} = 1$ and variance $\frac{1}{M}$. This means that mean value is not affected by number of antennas, but variance is reducing. If more and more antennas are added, the realization will be closer and closer to mean value as variance is reducing. As the consequence of spatial diversity, the squared norm of channel vector $\|g\|^2 = E\{\|g\|^2\}$, where $E\{\|g\|^2\}$ is the mean value. The probability that one antenna sees very bad or large channel realizations could be consequential. But when large set up is considered, independent channel realizations, will all start to behave in more deterministic manner. The squared norm of channel vector will be approximately mean value, when antennas are large. Another benefit as the consequence of beamforming gain is $\|g\|^2 \approx M$, when $M$ is large.

- **Asymptotic Motivation:** Consider two UAVs sending uplink signals $x_k$ for $K=1$ or 2. Channel $g_k = [g_k^1 \ldots g_k^M]^T \sim \text{CN} (0, I_M)$, noise $w \sim \text{CN} (0, I_M)$ and received signal $y = g_1 x_1 + g_2 x_2 + w$. When $y$ is being received, taking a linear detector i.e for UAV 1, $\hat{y}_1 = a_1^H y = a_1^H g_1 x_1 + a_1^H g_2 x_2 + a_1^H w$. Here, the signal remains $i.e \ a_1^H g_1 = \frac{g_1^1}{M} g_1 = \frac{\|g\|^2}{M} \xrightarrow{M \rightarrow \infty} E\{|g_1|^2\} = 1$, because of channel hardening property. Interference vanishes $i.e \ a_1^H g_2 = \frac{g_2^1}{M} g_1 \xrightarrow{M \rightarrow \infty} E\{|g_1|^2\} = 0$, because of favourable propagation property. Noise vanishes $i.e \ a_1^H w = \frac{g_1^1}{M} w \xrightarrow{M \rightarrow \infty} E\{|g_1|^2\} = 0$, because of favourable propagation property. Therefore, $\hat{y} = 1 + 0 + 0 = 1$ or $x_1$. This means for noise free and interference free communication $\hat{y} \xrightarrow{M \rightarrow \infty} x_1$.

### 5.4 Estimation of Channel Response

One of the challenges in massive MIMO enabled UAV communication is that in every coherence interval, communication channel needs to be learnt, to learn channel response. Channels are K UAVs with M length channel vectors. MK coefficients are to be estimated, to learn in each coherence interval. There is a requirement to learn because it is unknown to start with at transmitter and receiver.
Basic principle is to send known signal called pilot. This is what transmitter and receiver have predetermined in advance. Send this signal over the channel, check what is received and then detect what the channel is going to be. Since large coefficients are to be learned, it is important to be careful about design, about the way of sending pilots. As shown in Figure 11, when one pilot is used to estimate all coefficients, single antenna send one pilot signal, then it is going to be simultaneously received at all receive antennas i.e with one pilot all channels $g_1, g_2, \ldots, g_M$ can be estimated. When M pilots are used to estimate all coefficients, M antennas are sending pilots and M different pilots are needed to learn the channels. Therefore, it is number of transmit antennas that would be used to learn the channel from to determines number of pilots to be sent.

Figure 11

5.5 Time Division Duplexing (TDD) in Massive MIMO Enabled UAV Communication

There are different ways of dividing the time and frequency resources between uplink and downlink. One of the ways is TDD. In the Figure 12, each block is coherence interval, within one coherence interval uplink and downlink is sent, by switching between uplink and downlink. This is done fast enough so that channel stays fixed within one of these block. Therefore, in TDD, uplink and downlink are separated in time, K pilots are needed to learn all channels in system. Pilots to be sent need to be decided in uplink or downlink. In uplink there are K UAVs, therefore there are K pilots. In downlink there are M antennas at base station, therefore there are M pilots to learn channels. But good thing in TDD is that one can choose between uplink and downlink. In massive MIMO there are very
few UAVs/users than antennas at base station, which means system can be designed so that only K pilots are needed. In case of Frequency division duplexing (FDD), for every time uplink then send K pilots, for downlink send M pilots. Therefore system must support M pilots. Thus, to separate uplink and downlink in frequency, M pilots are needed.

5.6 Uplink Massive MIMO Enabled UAV Communication System Model

As shown in Figure 13, take time frequency resources and divide it into frames matched to coherence interval sizes. Frames are matched to coherence intervals with coherence time $T_C$ secs and
coherence bandwidth \(B_C\) Hz and channel interval \(\tau_c = B_C T_C\) complex samples. For analyzing the frames, one at a time, operation system is being broken. Therefore, in Uplink massive MIMO enabled UAV communication system Model, the received signal \(y\) is given as \(y = \sqrt{\rho_{ul}} Gx + w\). Write these signals in matrix form \(\begin{bmatrix} y_1 \\ \vdots \\ y_M \end{bmatrix} = \begin{bmatrix} g_{11}^1 & \ldots & g_{1K}^1 \\ \vdots & \ddots & \vdots \\ g_{M1}^M & \ldots & g_{MK}^M \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_K \end{bmatrix} \), \(w = \begin{bmatrix} w_1 \\ \vdots \\ w_M \end{bmatrix}\).

The parameters are normalized i.e Maximum power is \(\rho_{ul}\), \(x_1, \ldots, x_K\) has power \(\leq 1\) and channel of UAV \(K\) \(g_{k1}, \ldots, g_{MK} \sim \text{CN}(0, \beta_K)\) where \(\beta_K\) is large scale fading coefficient and normalized noise \(w_1, \ldots, w_M \sim \text{CN}(0, 1)\). Maximum SNR of UAV \(K\) is \(\rho_{ul} \beta_K\). Here, \(\rho_{ul} = (UL \text{ radiated power} \times \text{Antenna gain})/BN_0\), \(B\) is bandwidth and \(N_0\) is noise power spectral density. Here, \(y\) is \(M\) length vector, there are \(M\) receive antennas at base station. In matrix \(G\), each column is described for channel of one UAV to all antennas at base station and \(x\) is \(K\) length vector having signals from UAV 1 to UAV \(K\) along with known sequences of pilots. The way to estimate the channel is to consider not one vector \(x\) that is transmitted but multiple of \(x\), each one transmitted one after another.

Process of sending pilot sequences involves UAVs to send sequences of known information called pilot sequences or pilot signals eg for UAV 1 and UAV 2, the pilot signals are \(\emptyset_1\) & \(\emptyset_2\). These pilot sequences will be used by receiving base station in order to estimate the channel over which pilot sequence is transmitted. These sequences have length \(\tau_p\) and \(\emptyset\) length vector. Send pilot matrix \(\sqrt{\tau_p} \emptyset = \sqrt{\tau_p} [\emptyset_1 \ldots \emptyset_K]\) over \(\tau_p\) UAVs of the channel, \(y_p = \sqrt{\tau_p \rho_{ul}} G \emptyset H + w_p\). Here, \(\sqrt{\tau_p} \emptyset\) is pilot matrix, which is written with pilot sequences as \(\sqrt{\tau_p} [\emptyset_1 \ldots \emptyset_K]\), \(\tau_p\) is number of rows and \(K\) is number of columns. Send each row over channel of one particular UAV/user. Thus, write received signal over \(\tau_p\) channel users. The received signal \(y\) is a vector of length \(m\). Now, stack \(\tau_p\) of them as per columns to create \(y_p\). Each row is transmitting one at a time means every UAV is sending its individual pilot sequence.

When base station receives \(y_p\), the process of estimating the channel involve few steps. First step is to despread the pilot signal \(y_p' = y_p \emptyset = \sqrt{\tau_p \rho_{ul}} G \emptyset H \emptyset + w_p \emptyset\) by multiplying the received signal with pilot sequence \(\emptyset\). \(\sqrt{\tau_p \rho_{ul}}\) is constant, \(\emptyset H \emptyset\) is \(I_K\). \(G\) is required to be observed. The way of estimating guassian variable \(g\) in noise, is by considering \(y = \sqrt{P} g + w\), where \(P\) is constant, \(g \sim \text{CN}(0, \beta)\) and \(w \sim \text{CN}(0, 1)\). Mean square error is a way of estimating the channel, it is given by \(E[|\hat{g} - g|^2]\), where \(E\) is average or mean, \(\hat{g}\) is estimate and \(g\) is true value. In estimation theory when unknown variable \(g\) is observed, which is guassian distributed and it is observed in guassian noise then,
minimum mean square error (MMSE) estimator is $\hat{g} = E\{g|y\} = \frac{\sqrt{P}\beta}{1+P\beta} y$, where $P = \sqrt{\tau P\rho_{ul}}$. From this expression it is found that $\hat{g}$ is complex guassian distributed because $y$ is complex guassian distributed. Estimation error is $\tilde{g} = (\hat{g} - g) \sim CN \left(0, \beta - \frac{P\beta^2}{1+P\beta}\right)$, $\beta$ is original variance of $g$ and $\frac{P\beta^2}{1+P\beta}$ is variance of estimate. Estimate is $\tilde{g} \sim CN \left(0, \frac{P\beta^2}{1+P\beta}\right)$. Estimate of channels $[Y'_m]_{mk} = \sqrt{\tau P\rho_{ul}} g_k^m + [w_P]\varnothing_{mk}$, $m$ is row and $k$ is column, $\sqrt{\tau P\rho_{ul}}$ is constant, $[w_P]\varnothing_{mk}$ is noise, $g_k^m$ is what is to be estimated, and it is one complex guassian distributed channel coefficient between antenna $m$ at base station and UAV $k$. Therefore, MMSE estimate of $g_k^m$ from UAV $k$ to antenna $m$, is given by estimate $\hat{g}_k^m = E\{g_k^m | Y'_m\} = \frac{\beta_{k}}{1+\tau P\rho_{ul}\beta_k} [Y'_m]_{mk} \sim CN (0, \gamma_k)$, where $\gamma_k$ is variance of estimated channel. Estimation error is $\tilde{g}_k^m = \tilde{g}_k^m \sim CN (0, \beta_k - \gamma_k)$, where $\beta_k$ is variance of true channel and $\gamma_k$ is variance of estimated channel, $\gamma_k = \tau P\rho_{ul}\beta_k^2$. Mean square error (MSE) is given by $E\{|\tilde{g}_k^m - g_k^m|^2\} = E\{|\tilde{g}_k^m|^2\} = \beta_k - \gamma_k = \beta_k - \frac{\tau P\rho_{ul}\beta_k^2}{1+\tau P\rho_{ul}\beta_k}$, where $\tau P\rho_{ul} \to \infty$ i.e uplink power is very high or length of pilot sequence $\tau_P \to \infty$, because $\beta_k - \beta_k = 0$.

5.7 Performance of UAV Communication Uplink

![Figure 14](image)

Computation of exact capacity in different cases of point to point MIMO communication for UAVs was done assuming that the receiver knows the channel perfectly. But in practice the receiver can’t know the channel perfectly. Consider communication model shown in Figure 14, when receiver is forming the estimate $\hat{x}$, what does it have to access $y$ and channel information $\Omega$. This channel information is something around channel coefficient $g$. Earlier it was considered that the exact realization of $g$ is known even if it is a random number. But now generalize that because in practice the exact value is not known (have this estimation error). Thus, capacity can’t be computed but lower bound can be.
Capacity lower bound is \( C \geq E \left\{ \log_2 \left( 1 + \frac{\rho |E[g|\Omega|^2}{\rho \text{Var}[g|\Omega] + \text{Var}[w|\Omega]} \right) \right\} \). Here, SNR is 
\[
\frac{\rho |E[g|\Omega|^2}{\rho \text{Var}[g|\Omega] + \text{Var}[w|\Omega]},
\]
\( \rho \) is power, \(|E[g|\Omega]|^2 \) is absolute value square of the channel but here it is not actual \( g \) but estimate of \( g \) given \( \Omega \). \( \rho \text{Var}[g|\Omega] \) is uncertainty around \( E[g|\Omega] \) and \( \text{Var}[w|\Omega] \) is variance of noise. Since fast fading channel is considered, the realization of the channel will be changing all the time. Then this will be a random number \( \log_2(1 + \text{SNR}) \). Therefore, expectation \( E \) is taking expectation over different channel realization. The capacity is usually smaller than \( C \), and it is equal when there is perfect knowledge of channel.

For uplink data transmission, the received signal \( y \) is given as \( y = \sqrt{\rho_{ul}} Gx + w \). The signals \( x_k = \sqrt{\eta_k} q_k \), where, \( q_k \sim \text{CN}(0,1) \) is data symbol having complex Gaussian variance 1, and \( 0 \leq \eta_k \leq 1 \) controls the power. Each signal is divided into \( x_k \). \( \eta_k \) is power control coefficient, to figure out if the UAV will be transmitting with full power (1) or zero/no power (0). Keep \( \eta_k \) as constant. Channel of UAV \( k \) \( g_k^1 \ldots g_k^m \sim \text{CN}(0,\beta_k) \) and \( w \sim \text{CN}(0,\sigma_w) \)

For Linear receiver processing, rewrite this model without using \( x \) but using only \( \eta_k \) and \( q_k \). Therefore, the received signal \( y \) is now given as \( y = \sqrt{\rho_{ul}} GD_{\eta}^{1/2} q + w \), where \( \sqrt{\eta_k} \sim \sqrt{D_{\eta}} \).

\[
D_{\eta} = \begin{pmatrix} \eta_1 & 0 & 0 \\ 0 & \ddots & 0 \\ 0 & 0 & \eta_k \end{pmatrix} q = \begin{pmatrix} q_1 \\ q_k \end{pmatrix}.
\]
Receiver would like to receive \( y \) and guess each of the transmitted signals \( q_1 \ldots q_k \). Let's find out for one particular UAV \( i \). Select a receiver filter \( a_i \) for UAV \( i \) such that

\[
a_i^H y = \sqrt{\rho_{ul}} \ a_i^H D_{\eta}^{1/2} q + a_i^H w = \Sigma_{k=1}^k a_i^H g_k \sqrt{\rho_{ul} \eta_k} q_k + a_i^H w \approx q_i.
\]
Therefore by multiplying with \( a_i^H \), \( q_i \) is obtained, as for \( k = i \), \( a_i^H g_k \sqrt{\rho_{ul} \eta_k} \) becomes 1 and for interfering signals from other UAVs, this term is desired to be 0, along with noise term to be 0. In order to figure out good way of selecting \( a_i \), end performance should be kept in mind namely capacity lower bound \( C \geq E \left\{ \log_2 \left( 1 + \frac{\rho |E[g|\Omega|^2}{\rho \text{Var}[g|\Omega] + \text{Var}[w|\Omega]} \right) \right\} \), \( a_i \) should be selected to maximize \( C \). Thus, each term needs to be computed. Here, \( \rho, g \) & \( \Omega \) have different meanings. \( \rho = \rho_{ul} \eta_i, g = a_i^H g_i, x = q_i \) and \( \Omega = \{ \hat{g}_1, \ldots, \hat{g}_k \} \) MMSE estimates. \( w' = \Sigma_{k=1,k\neq i}^k a_i^H g_k \sqrt{\rho_{ul} \eta_k} q_k + a_i^H w \), having all of the other added term excluding \( k = i \) and noise term. Numerator \( E[g|\Omega] = E[a_i^H g_i|\hat{g}_1, \ldots, \hat{g}_k] = a_i^H E[\hat{g}_i|\hat{g}_1, \ldots, \hat{g}_k] - a_i^H E[\hat{g}_i|\hat{g}_1, \ldots, \hat{g}_k] = a_i^H \hat{g}_i \). Here, \( \hat{g}_i \) is expected value of estimate and \( \hat{g}_i \) is expected value of estimation error. \( g_i = \hat{g}_i - \hat{g}_i \) and \( E[\hat{g}_i|\hat{g}_1, \ldots, \hat{g}_k] = E[\hat{g}_i] = 0. \) \( a_i \) is selected based on
\( \Omega = \{ \hat{g}_1, \ldots, \hat{g}_k \} \) and \( g = a_i^T g_i \). First term of the denominator \( \text{Var}\{g|\Omega\} = E\{|g|^2|\Omega\} - |E\{g|\Omega\}|^2 \). Here, \( E\{|g|^2|\Omega\} = \{ |a_i^T g_i|^2|\Omega\} = a_i^H E\{ \hat{g}_i \hat{g}_i^H \} + \hat{g}_i \hat{g}_i^H - \hat{g}_i \hat{g}_i^H |\Omega\}a_i = a_i^H (\hat{g}_i \hat{g}_i^H + (\beta_i - \gamma_i)I_M - 0 - 0) a_i = a_i^H (\hat{g}_i \hat{g}_i^H + (\beta_i - \gamma_i)I_M) a_i \). Second term of denominator \( \text{Var}\{w'|\Omega\} = \text{Var}\{ \sum_{k=1, k \neq i}^k a_i^H g_k \sqrt{\rho_{ul} \eta_k} q_k + a_i^H w |\Omega\} \). \( E\{w'|\Omega\} = 0 \), as \( E\{q_k\} \) and \( E\{w\} = 0 \). Therefore,

\[
\text{Var}\{w'|\Omega\} = E\{|w'|^2|\Omega\} = \sum_{k=1, k \neq i}^k E\{ |a_i^H g_k|^2 |\Omega\} \sqrt{\rho_{ul} \eta_k} E\{|q_k|^2 |\Omega\} + E\{ |a_i^H w|^2 |\Omega\} = \sum_{k=1, k \neq i}^k a_i^H (\hat{g}_k \hat{g}_k^H + (\beta_k - \gamma_k)I_M) a_i \sqrt{\rho_{ul} \eta_k} + a_i^H I_M a_i \). Putting all these terms in main equation \( C \geq E\left\{ \log_2 \left( 1 + \frac{\rho_{ul} |a_i^H \hat{g}_i|^2}{|a_i^H b_i|^2} \right) \right\} \), where \( B_i = \sum_{k=1, k \neq i}^k \rho_{ul} \eta_k \hat{g}_k \hat{g}_k^H + \sum_{k=1}^k \rho_{ul} \eta_k (\beta_k - \gamma_k) I_M + I_M \). Here, the term \( \frac{\rho_{ul} |a_i^H \hat{g}_i|^2}{|a_i^H b_i|^2} \) has the mathematical structure similar to generalized Rayleigh quotient \( |a^H b|^2 / a^H Ba \). For given \( b \) vector and \( B \) matrix, the ratio is maximized by \( a = B^{-1}b \). Considering that \( B \) is changed to identity matrix \( I_M \) or is removed, the ratio is maximized by \( a = b \). The extra term \( B^{-1} \) is whitening. When there is no interference or estimation error, then \( b \) is going to be an identity matrix and we get maximum ratio combining. When there is interference by UAVs then, we would like to whiten the interference and noise terms. Thus, \( |a^H b|^2 / a^H a \) is maximum when \( a \& b \) are same and they point in same direction. This is what is done in maximum ratio combining i.e selecting receive filter equal to channel. Now use this result to maximize the capacity lower bound equation.

### 5.7.1 Maximizing the Uplink Capacity Lower Bound

\[
C \geq E\left\{ \log_2 \left( 1 + \frac{|a_i^H b_i|^2}{|a_i^H B_i a_i|^2} \right) \right\}, \quad \text{where} \quad b_i = \sqrt{\rho_{ul} \eta_i} \hat{g}_i \quad \text{and} \quad B_i = \sum_{k=1, k \neq i}^k \rho_{ul} \eta_k \hat{g}_k \hat{g}_k^H + \sum_{k=1}^k \rho_{ul} \eta_k (\beta_k - \gamma_k) I_M + I_M. \]

Maximize this by selecting \( a_i = B^{-1} b_i = \sqrt{\rho_{ul} \eta_i} B_i^{-1} \hat{g}_i \); this is minimum mean square error (MMSE) combining i.e minimizing mean square difference between inner product and \( q_i \), because \( a_i^H y = q_i \) is desired. \( \hat{g}_i \) is the channel estimate of the desired channel \( \hat{g}_i \) in particular direction. While selecting receive filter, \( a_i \) can be placed anywhere, but if it is put in alignment or in line with channel estimate of the desired channel, then the gain of the channel can be maximized i.e numerator of SNR expression. But that is not an optimal thing to do. Optimal thing is to make sure that \( a_i \) points to somewhere in between zero and channel estimate direction i.e take
\( \hat{g}_1 \) and rotate it using \( B_i^{-1} \). One can also select pointing orthogonally. Thus, the denominator of SNR expression can be minimized.

### 5.7.2 Maximum Ratio Processing

As in uplink massive MIMO enabled UAV communication system, the sum capacity with \( K=2 \) and channel matrix \( G = [g_1 g_2] \) is

\[
R_1 + R_2 = \log_2 \left( \det(I_2 + \rho_{ul} G^H G) \right) = \log_2 \left( I_2 + \rho_{ul} \begin{bmatrix} \|g_1\|^2 & g_1^H g_2 \\ g_2^H g_1 & \|g_2\|^2 \end{bmatrix} \right),
\]

where \( I_2 \) is identity matrix of size 2 or UAVs 2, \( \rho_{ul} \) is SNR, \( G^H G \) is product of channel matrix and hermitian transpose of itself, \( \|g_1\|^2 \) is squared norm of one of the channel vector, \( g_1^H g_2 \) is inner product of two channel vectors and \( \|g_2\|^2 \) is squared norm of other channel vector. In this case since \( M \) antennas are there at base station thus all vectors are \( m \) dimensional. Thus, the equation becomes

\[
R_1 + R_2 = \log_2 \left( 1 + \rho_{ul} \|g_1\|^2 \right) \left( 1 + \rho_{ul} \|g_2\|^2 \right) - \rho_{ul}^2 |g_1^H g_2|^2 \leq \log_2 \left( 1 + \rho_{ul} \|g_1\|^2 \right) + \log_2 \left( 1 + \rho_{ul} \|g_2\|^2 \right),
\]

here each term is the point to point capacity of individual UAV. Whenever the inner product between two vectors \( g_1^H g_2 \) is zero i.e both vectors are orthogonal, larger sum capacity is achieved. Therefore, it is preferred to have channel vectors orthogonal to each other so that both UAVs get maximum capacity at the same time i.e the capacity region is square. This is called favourable propagation. A collection of channel vectors \( \{g_k\} \) are said to offer favourable propagation if \( g_k^H g_i = 0 \) for \( k,i=1,...,K, k\neq i \). Therefore, \( K \) UAVs can communicate as if they are alone in system even if they are transmitting at the same time and frequency. This is because channel vectors are orthogonal, which allows their base stations to separate them easily in space. In practice it is never satisfactory, but there is one thing that is satisfied called asymptotic favourable propagation. It is

\[
\frac{1}{M} g_k^H g_i \rightarrow 0 \quad \text{as} \quad M \rightarrow \infty \quad k,i=1,...,K, k\neq i.
\]

This means when more antennas are added, array becomes larger, have smaller beam width that means sending signals into smaller parts of space, directive signals more and more. If UAVs are not at same location, then add more antennas to make them easily separable and above equation get satisfied.

Consider a sequence \( x_1, x_2, ... \) of independent and identically distributed random variables. Assume \( E[X_i] = \mu \) for \( i=1,2,... \) and \( Var\{X_i\} = \sigma^2 < \infty \) for \( i=1,2,... \). Then the sample average \( \bar{X}_n = \left( X_1 + X_2 + ... + X_n \right)/n \) converges to the expected value \( \bar{X}_n \rightarrow \mu \) as \( n \rightarrow \infty \). Variance

\[
Var\{\bar{X}_n\} = \frac{(Var\{X_1\} \times ... \times Var\{X_n\})}{n^2} = \frac{n \sigma^2}{n^2} = \sigma^2/n.
\]

Law of large numbers can be used to analyse the properties of Rayleigh fading channels. Channels are independently distributed as \( g_k \sim CN(0, \beta_k I_M) \).

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As more antennas are added, sequence in vector \( g_k \), gets longer i.e having more and more terms. This arrangement offers both channel hardening and favourable propagation. When squared norm of the channel vector is computed and divided with number of terms \( (M) \). Then, this is sample average of the absolute value square for this vector \( \langle g_k \rangle \). When more and more terms are added in the vector, law of large numbers can be applied. \( \frac{1}{M} \| g_k \|^2 \to \beta_k \) for \( M \to \infty, k = 1, \ldots k \), where \( \beta_k \) is the mean value of each of the individual absolute value squares and that is equal to its variance. This is called channel hardening, which is consequence of diversity gain. Therefore, when more and more antennas are added, deterministic channel gain is obtained. It offers asymptotic favourable propagation as \( \frac{1}{M} g_k^H g_i \to 0 \), for \( M \to \infty, k = 1, \ldots k & k \neq i \). When summation is done over \( M \) terms, sample average converges to mean. Therefore, for Rayleigh fading channels, the approximations when \( M \) is large are given by
\[
\frac{1}{M} \| g_k \|^2 \approx \beta_k \quad \text{and} \quad \frac{1}{M} g_k^H g_i \approx 0.
\]
Now, use this property to design signal processing in much simpler way.

The problem is that channel is not known, instead the estimate of the channel is known only at the receiver side. Estimate is \( \hat{g}_k^m = E\{g_k^n|Y_p^r\} = \frac{\beta_k}{1+\tau_{p,ul}^2 \rho_k} [Y_p^r]_{mk} \sim \text{CN}(0, \gamma_k) \), where \( \gamma_k \) is variance of estimated channel. Estimation error is \( \hat{g}_k^m = g_k^m - \gamma_k^m \sim \text{CN}(0, \beta_k - \gamma_k) \), where \( \beta_k \) is variance of true channel and \( \gamma_k \) is variance of estimated channel. \( \gamma_k = \frac{\tau_{p,ul}^2 \rho_k}{1+\tau_{p,ul}^2 \rho_k} \). Putting these in vector notation,
\[
\hat{g}_k = \begin{bmatrix} \hat{g}_k^1 \\ \vdots \\ \hat{g}_k^M \end{bmatrix} \sim \text{CN}(0, \gamma_k I_M), \text{where is the estimated channel from antenna 1 to M with respect to UAV k.}
\]

All have same variance \( \gamma_k \). \( \hat{g}_k = \begin{bmatrix} \hat{g}_k^1 \\ \vdots \\ \hat{g}_k^M \end{bmatrix} \sim \text{CN}(0, (\beta_k - \gamma_k) I_M) \). Therefore, considering the estimated channel is independently distributed as \( \hat{g}_k \sim \text{CN}(0, \gamma_k I_M) \), as per property of Rayleigh fading channel, it offers channel hardening i.e \( \frac{1}{M} \| \hat{g}_k \|^2 \to \gamma_k \) for \( M \to \infty, k = 1, \ldots k \). It also offers asymptotic favourable propagation i.e \( \frac{1}{M} \| \hat{g}_k \|^2 \to \gamma_k \) for \( M \to \infty, k = 1, \ldots k & k \neq i \). Therefore, the approximations when \( M \) is large are given by \( \frac{1}{M} \| \hat{g}_k \|^2 \approx \gamma_k \) & \( \frac{1}{M} \hat{g}_k^H \hat{g}_i \approx 0 \). As we know Capacity lower bound is given by \( C \geq E \left\{ \log_2 \left( 1 + \frac{\rho|E(g|\Omega)|^2}{\rho\text{VAR}(g|\Omega) + \text{VAR}(W|\Omega)} \right) \right\} \). Let’s now particularize this for the case when \( g \) is a constant deterministic channel. Deterministic and known channel coefficient \( g(\Omega = \{g\}) \) i.e expected value of \( g \) given \( \Omega \) is \( g \), \( E\{g|\Omega\} = g \). Thus, \( C \geq E \left\{ \log_2 \left( 1 + \frac{\rho|E(g|\Omega)|^2}{\rho\text{VAR}(g|\Omega) + \text{VAR}(W|\Omega)} \right) \right\} = \log_2 \left( 1 + \right) \)
This is because expectation is no more required as the channel is known, \( \text{Var}(g) = 0 \) as when \( g \) is known \( \Omega = \{g\} \). Let us now rewrite uplink signal by utilizing this expression.

\[
\rho\|g\|^2 \over \text{Var}(w) \].
\]

5.7.3 Uplink Capacity Lower Bound: UAV Communication

The received uplink signal is given by \( y = \sum_{k=1}^{k} g_k \sqrt{\rho_{ul} \eta_k} q_k + w = \sum_{k=1}^{k} \hat{g}_k \sqrt{\rho_{ul} \eta_k} q_k - \sum_{k=1}^{k} \hat{g}_k \sqrt{\rho_{ul} \eta_k} q_k + w \), where summation is sum of all signals transmitted by all UAVs, \( q_k \) is data signal and \( g_k \) is true channel. \( \sum_{k=1}^{k} \hat{g}_k \sqrt{\rho_{ul} \eta_k} q_k \) is the channel estimate and is useful part of the equation as receiver knows the channel estimate & that gives it a possibility of extracting information with \( q_k \). \( \sum_{k=1}^{k} \hat{g}_k \sqrt{\rho_{ul} \eta_k} q_k + w \) is estimation error and is unusable part as estimation error is not known, noise is not known, data signal is not known, thus unusable and shown as \( w' \). Now assign a receiver filter \( a_i \) for UAV \( i \). This is detection vector or combining vector. \( a_i^H y = a_i^H \hat{g}_i \sqrt{\rho_{ul} \eta_i} q_i + \sum_{k=1,k \neq i}^{k} a_i^H \hat{g}_k \sqrt{\rho_{ul} \eta_k} q_k + a_i^H w' \), where \( a_i^H \hat{g}_i \sqrt{\rho_{ul} \eta_i} q_i \) is desired part (\( k=i \)) from where information is to be extracted, \( \sum_{k=1,k \neq i}^{k} a_i^H \hat{g}_k \sqrt{\rho_{ul} \eta_k} q_k \) is interference (all UAVs except \( k=i \)). Desired part is to be made as large as possible. Consider \( \hat{g}_i \sim CN(0, \gamma_i I_M) \), which value of \( a_i \) maximizes the ratio \( \|a_i^H \hat{g}_i\| \over \|a_i\| \). Use Cauchy Schwartz Inequality \( \|a_i^H \hat{g}_i\| \leq \|a_i\| \|\hat{g}_i\| \), with equality of \( a_i = c \hat{g}_i \) for some constant \( c \neq 0 \). Cauchy Schwartz Inequality also says that largest value is achieved when two multiplying complex vectors happen to be parallel i.e \( a_i = c \hat{g}_i \). Therefore, for making desired part of received signal as large as possible, one should select receiver filter to be equal to channel estimate \( \hat{g}_i \) of same user multiplied with non zero constant and this is called MR processing. Thus, \( a_i = c \hat{g}_i \) is called maximum ratio processing, same as MRC in point to point MIMO for \( c = 1/\|\hat{g}_i\| \).

For calculating received signal when using MR processing, let us put \( a_i = \frac{1}{M} \hat{g}_i \), we get \( a_i^H y = \frac{g_i^H \hat{g}_i}{M} \sqrt{\rho_{ul} \eta_i} q_i + \sum_{k=1,k \neq i}^{k} \frac{g_k^H \hat{g}_k}{M} \sqrt{\rho_{ul} \eta_k} q_k + \frac{g_i^H w'}{M} \), where \( \frac{g_i^H \hat{g}_i}{M} \approx \gamma_i \) because of channel hardening, \( \frac{g_k^H \hat{g}_k}{M} \approx 0 \) because of favourable propagation and \( \frac{g_i^H w'}{M} \approx 0 \). Two things happen here, firstly the interference and noise terms are small when large number of antennas are there, secondly even if there is fading channel, after MR processing is applied, the desired signal term would be approximately equal to \( q_i \), which is deterministic number.
Applying Use and Forget technique i.e using channel estimate to compute the receiver filter

\[ a_i = \frac{1}{M} \hat{g}_i, \] apply it or use it and it is found the term \( \frac{g_i^H \hat{g}_i}{M} \) is deterministic. Therefore, channel estimate need not to be remembered anymore. Thus, \( a_i^H y = y_i \sqrt{\rho_{ul} \eta_i q_i} + \left( \frac{g_i^H \hat{g}_i}{M} - y_i \right) \sqrt{\rho_{ul} \eta_i q_i} + \sum_{k=1, k \neq i}^k \frac{g_k^H \hat{g}_k}{M} \sqrt{\rho_{ul} \eta_k} q_k + g_i^H \hat{g}_i \sqrt{\rho_{ul} \eta_i} w' \), where first term \( y_i \sqrt{\rho_{ul} \eta_i q_i} \) is the desired part with deterministic channel and other terms are uncorrelated interference and noise \( w \). Capacity bound with deterministic channel having desired signal \( x = q_i \), transmit power \( \rho = \rho_{ul} \eta_i \) and deterministic & known channel coefficient \( g = y_i \), is given by \( C \geq \log_2 \left( 1 + \frac{\rho |g|^2}{\text{Var}(w)} \right) \), where \( \rho |g|^2 = \rho_{ul} \eta_i y_i^2 \) and \( \text{Var}(w) = \frac{1}{M} (\sum_{k=1}^k \rho_{ul} \eta_k \beta_k + 1) \). Therefore, capacity lower bound with MR processing and use & forget technique is given by

\[ C \geq \log_2 \left( 1 + \frac{M \rho_{ul} \eta_i y_i}{\sum_{k=1}^k \rho_{ul} \eta_k \beta_k + 1} \right) \]

This is a closed form expression meaning no expression left to compute. In the numerator, the coherent beam gain grows with antennas M, power \( \rho_{ul} \eta_i \) and estimation equality \( y_i \). The denominator includes the sum of non coherent interference from all UAVs plus noise variance, 1 is noise variance normalized to 1 and \( \sum_{k=1}^k \rho_{ul} \eta_k \beta_k \) is the interference term containing summation of all UAVs with their transmit power \( \rho_{ul} \eta_k \) and variance of channel \( \beta_k \). The interference appears to be large because of full transmit power and actual variance of channel not something that depends on estimates. The important thing is that the interference is not scaled with number of antennas and is called non coherent interference. Therefore, when more antennas are used to receive things, the desired signal is amplified, but the interference term is not amplified. As number of antennas are increased, interference term remain the same while numerator increases with number of antennas. There is no need to focus on interference as number of antennas increase.

### 5.8 Performance of UAV Communication Downlink (DL)

For describing downlink system model for massive MIMO enabled UAV communication, consider k UAVs as shown in Figure 8. For UAV i, there is channel vector of \( g_i \) of length m i.e

\( g_i = \begin{bmatrix} g_{i1}^1 \\ \vdots \\ g_{iM}^M \end{bmatrix} \). Since there are M antennas at base station, m dimensional signals are sent. Signal sent by base station is \( \sqrt{\rho_{dl}} x \), where \( \sqrt{\rho_{dl}} \) is transmitted power and \( x \) is m dimensional vector. The received
signal at UAV $i$ is given by $y_i = \sqrt{\rho_{dl}} g_i^T x + w_i$, where $\sqrt{\rho_{dl}} x$ is transmitted signal, $g_i$ is channel vector to users and additive noise $w_i \sim \mathcal{CN}(0,1)$. Write these in matrix form $y = \sqrt{\rho_{dl}} G^T x + w$ such that

$$
\begin{bmatrix}
y_1 \\
y_2 \\
\vdots \\
y_k
\end{bmatrix}
= 
\begin{bmatrix}
g_1^T \\
g_2^T \\
\vdots \\
g_k^T
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2 \\
\vdots \\
x_M
\end{bmatrix} + 
\begin{bmatrix}
w_1 \\
w_2 \\
\vdots \\
w_k
\end{bmatrix}
$$

In $G$ channel matrix, each column is one of the channel from UAV to base station. This is same matrix as uplink but with transpose i.e inner product between $g_i$ conjugate and $x$ because each of the transmitted signal is multiplied by corresponding channel coefficient in $g_i$, so there is no complex computation here. The parameters are normalized i.e maximum power is $\rho_{dl}$ $E\{\|x\|^2\} \leq 1$, channel of the user $k$ $g_k^1 \ldots g_k^M \sim \mathcal{CN}(0, \beta_k)$ and normalized noise $w \sim \mathcal{CN}(0, I_k)$.

### 5.8.1 Linear Precoding

Select the signal $x$ that base station is going to transmit and its containing signals are supposed to be sent to all users i.e for UAV $k$, the message symbol is $q_k$. Now, select the transmitted signal $x = \sum_{k=1}^{K} \sqrt{\eta_k} a_k q_k$ where, message symbol to UAV $k$ is $q_k$, $E\{|q_k|^2\} = 1$ having zero mean, precoding vector $a_k$, $E\{\|a_k\|^2\} = 1$ and power control coefficient $\eta_k \leq 1$. There are two different terms $a_k$ & $\eta_k$. There is a need to represent this scalar number $q_k$ data symbol and map them to $M$ different signals ($x$) in order to determine how signals are transmitted to different UAVs. That is where $m$ dimensional precoding vector $a_k$ is used. Thus, $a_k$ is precoding vector of length $m$ which tells the direction of signal sent to UAV $k$. It is ensured that precoding vector is normalized i.e $E\{\|a_k\|^2\} = 1$, squared norm on the average is 1. Since $a_k$ is selected based on the estimates of random channels and is also a random variable, therefore average is needed.

The power needs to be allocated to different UAVs at base station. Therefore, power control coefficient $\eta_k \leq 1$ and in addition to this, the total power at base station needs to be equal to $\rho_{dl}$ i.e $E\{|x|^2\} \leq 1$. In addition to each of the $\eta_k$ parameter $\leq 1$, actually the summation $\leq 1$, this is the power constraint that matters. Total power constraint is $\sum_{k=1}^{K} \eta_k \leq 1$. If total power for downlink is $\rho_{dl}$, then $\eta_k$ parameter tells how the power is divided among UAVs and it is desired to have equal power to all UAVs $\eta_k = 1/k$, using whole power budget. Fixed value of $\eta_k$, is considered, but it can also be optimized.
5.8.2 Downlink Capacity Lower Bound: UAV Communication

To identify how much data can be transmitted to different UAVs, first check the capacity bound with deterministic channel as shown in Figure 14, having desired signal $x$, transmit power $\rho$ and deterministic channel coefficient $g$ known at receiver. Capacity is $C \geq \log_2 \left(1 + \frac{\rho |g|^2}{\text{Var}(w)} \right)$, the received downlink signal is $y = \sqrt{\rho_d} g^T x + w$, where $x = \sum_k \sqrt{\eta_k} a_k q_k$, total transmitted power is $\rho_d$, division of the power between UAVs is denoted by $\eta_k$, precoding vector for UAV $k$ is $a_k$ and data transmitted towards UAV $k$ is $q_k$. Thus, $y_i = g_i^T \left( \sum_{k=1} \sqrt{\eta_k} a_k q_k \right) + w_i = \sqrt{\rho_d \eta_i} g_i^T a_i q_i + \sum_{k=1, k \neq i} \sqrt{\rho_d \eta_k} g_i^T a_k q_k + w_i$. Here, first term $\sqrt{\rho_d \eta_i} g_i^T a_i q_i$ is the desired signal and balance term is interference & noise. Problem is that receiver does not know $g_i^T a_i$. There is no mechanism in downlink for UAVs to know channels. No pilots are sent, as it is not necessary. Important thing is pilots are sent in one direction i.e in uplink so that channel estimation can be done and select precoding vector in such a way that inner product $g_i^T a_i$ becomes predictable at receiver. The meaning of that could be different in different context. But when there are many antennas, inner product as precoding vector $g_i^T a_i \approx E \{ g_i^T a_i \}$ is known, where $a_i$ is selected as function of estimate of channel vector which is approximately equal to its mean value when antennas are large. For example $a_i = g_i^T$, then $g_i^T a_i = |g_i|^2$ is equal to mean value because of channel hardening. The mean value is not random, it can be assumed that user knows this mean value, even if it does not know the particular realization. Therefore, use this expression $E \{ g_i^T a_i \}$ in received signal expression. Therefore, add and subtract expected value, which is similar to use and forget technique used in uplink. $y_i = \sqrt{\rho_d \eta_i} g_i^T a_i q_i + \sum_{k=1, k \neq i} \sqrt{\rho_d \eta_k} g_i^T a_k q_k + w_i$. $y_i = \sqrt{\rho_d \eta_i} E \{ g_i^T a_i \} q_i + \sqrt{\rho_d \eta_i}(g_i^T a_i - E \{ g_i^T a_i \}) q_i + \sum_{k=1, k \neq i} \sqrt{\rho_d \eta_k} g_i^T a_k q_k + w_i$, where first term is desired signal and balance terms are interference & noise. As the capacity is given by $C \geq \log_2 \left(1 + \frac{\rho |g|^2}{\text{Var}(w)} \right)$, capacity lower bound with any precoding is given by $C \geq \log_2 \left(1 + \frac{\rho_d |g|^2}{\text{Var}(w)} \right)$. In this expression as expectations are used, take average over small scale fading. Numerator is proportional to $E \{ g_i^T a_i \}$ squared and denominator has sum of interferences proportional to $E \{ |g_i^T a_k|^2 \}$ from all UAVs plus noise variance. More the channel hardening, closer this will be to true capacity i.e expectation value will be closer to actual realization achieved in coherence interval. No approximators are involved here, this expression can be used to find capacity lower bound for any precoding vector.
Choice of precoding vector depends on few factors. In uplink, optimal type of receiver filter was derived (MMSE combining). It is easier in uplink because in uplink expression of capacity lower bound, SNR had one combining vector and it all depend on one combining vector. This is because base station is receiving signals from UAVs, one of the transmitting signal was differentiated from interference & noise and thus one filter for one UAV was selected. But in capacity lower bound for downlink, all precoding vectors are in the expression i.e $a_i$ for UAV i and $\sum a_k$ as sum of all UAVs. Therefore, all precoding vectors are to be selected jointly somehow and the choice made for one UAV will affect other UAV as well. So one choice of $a_i$ might be focusing on one UAV and make its numerator as large as possible but it might also create lot of interference in terms of other UAVs.

For selection of precoding vector, recall uplink processing. For MMSE receiver $a_i = \sqrt{\frac{\rho_{ul}}{\eta_i B_i^{-1}}} \hat{g}_i$, and MR receiver $a_i = \hat{g}_i$ where $a_i$ is receiver filter, $B_i^{-1}$ contains channel estimates of all other UAVs and noise along with estimation errors, $\hat{g}_i$ contains channel estimate of UAV i.

Therefore, suggested precoding principle is to transmit in the direction where UAV is heard most clearly. Considering what was a good uplink processing, in which direction vectors should be pointed, so that good trade off between having strong signal and interference is obtained. So when transmission is being done back to the UAV, how we are processing uplink signal from corresponding UAV, should be considered and use that as guideline. Therefore, downlink precoding should be selected to be equal to corresponding uplink processing. Thus, downlink precoding scheme MMSE precoder $a_i = c_i \sqrt{\rho_{ul}} B_i^{-1} \hat{g}_i$ where $c_i = \frac{1}{\sqrt{E[\|\rho_{ul} B_i^{-1} \hat{g}_i\|^2]}}$. Earlier $a_i$ was considering about power. In downlink the power is considered, $a_i$ need to have squared norm which on the average is equal to 1. Therefore, taking scaling factor $c_i$ and compute the expected value of the squared norm of rest of it. Thus, making sure that $a_i$ has norm which is equal to 1. Any of the precoding vector can be taken and put into capacity lower bound expression. Then compute the expectation to get capacity lower bound. In MMSE, the expectation can not be computed exactly, there is a need to run the simulation to approximate them. But in MR, expectation expression can be computed, but need to remember how to select the channel estimate.

For MMSE estimate of $g_k^m$ from user k to antenna m. Estimate: $\hat{g}_k^m = E\{g_k^m | Y_p\} = \frac{\sqrt{\tau_p \rho_{ul} \beta_k}}{1 + \tau_p \rho_{ul} \beta_k} [Y_p]_{mk} \sim \mathcal{CN}(0, \gamma_k)$ where $\gamma_k = \frac{\tau_p \rho_{ul} \beta_k}{1 + \tau_p \rho_{ul} \beta_k}$. Estimation error: $\tilde{g}_k^m = g_k^m - \hat{g}_k^m \sim \mathcal{CN}(0, \beta_k - \gamma_k)$. 

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5.8.3 Downlink Capacity Lower Bound with MR Processing

\[
C \geq \log_2 \left( 1 + \frac{M \rho dl \eta_i \gamma_i}{\sum_{k=1}^{l} \rho dl \eta_k \beta_i + 1} \right) \geq \log_2 \left( 1 + \frac{M \rho dl \eta_i \gamma_i}{\beta_i \sum_{k=1}^{l} \rho dl \eta_k + 1} \right),
\]

here numerator \( M \rho dl \eta_i \gamma_i \) is the desired signal where coherent beamforming gain grows with antennas \( M \), power \( \rho dl \eta_i \) and estimation equality \( \gamma_i \). Denominator \( \sum_{k=1}^{l} \rho dl \eta_k \beta_i + 1 \) is sum of non coherent interference plus noise variance which does not grow with number of antennas. The interference between UAVs does not grow with number of antennas. Therefore, it is beneficial to have large number of antennas because desired signal get beamforming gain but not interference term.

Uplink capacity lower bound: \( C \geq \log_2 \left( 1 + \frac{M \rho ul \eta_i \gamma_i}{\sum_{k=1}^{l} \rho ul \eta_k \beta_k + 1} \right) \)

Downlink Capacity lower bound: \( C \geq \log_2 \left( 1 + \frac{M \rho dl \eta_i \gamma_i}{\beta_i \sum_{k=1}^{l} \rho dl \eta_k + 1} \right) \)

Uplink and downlink expressions of capacity lower bound have same structure. However, the differences are in terms of uplink and downlink interference. Uplink interference is from UAVs \( (\beta_1 \cdots \beta_k) \) i.e interference from UAV \( k \) goes through the channel of UAV \( k \) to the base station. Downlink interference is from base station \( (\beta_i) \) i.e all interference is from same point or base station, even if the signals are meant for another UAV, it is going to go through the channel from base station to UAV \( i \) that is being considered. Thus, \( \beta_i \) comes before summation term. Uplink interference comes from different UAVs through different channels to base station. In downlink, all interference is coming through the same channel from base station.

Channel capacity is the performance metric for analyzing any communication system. Uplink and downlink Channel capacities of Massive MIMO enabled UAV communications, have been evaluated in this paper. This will help in further study and research on specific issues of enhancing UAV communications.

6. Numerical Results and Key Insights

We now carry out the performance analysis of uplink of UAVs by considering a single cell scenario having cell area of approximately 0.25 Km x 0.25 Km with one base station. The base station has the capability of mounting antennas as per selection of different antenna techniques. The values of various parameters considered in the set up are given in Table 1 below.
Table 1

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of cells</td>
<td>1</td>
</tr>
<tr>
<td>Number of base stations</td>
<td>1</td>
</tr>
<tr>
<td>Number of antennas per base station</td>
<td>M</td>
</tr>
<tr>
<td>Number of UAVs per cell (K)</td>
<td>2</td>
</tr>
<tr>
<td>Channel gain ($\beta$)</td>
<td>-100 dB</td>
</tr>
<tr>
<td>Noise variance ($N_o$)</td>
<td>-80 dBm</td>
</tr>
<tr>
<td>Uplink transmit power</td>
<td>20 dBm</td>
</tr>
<tr>
<td>Length of pilot sequences ($\tau_p$)</td>
<td>10</td>
</tr>
<tr>
<td>Variance of true channel ($\beta_i$)</td>
<td>10 dB</td>
</tr>
<tr>
<td>Power control coefficient ($\eta_i$)</td>
<td>1</td>
</tr>
</tbody>
</table>

6.1 Effect of different Antenna Techniques on Performance of UAV Communication Links

Various antenna techniques considered with their base station antenna configuration are SIMO having single antenna, point to point MIMO having 2 antennas, MU-MIMO having 8 antennas and massive MIMO having 100 antennas. Performance analysis of communication link of UAV has been carried out by comparing channel capacity of these antenna techniques. Figure 15 shows that channel capacity increases with increase in antennas at base station. Massive MIMO having 100 antennas at base station provides maximum channel capacity.

![Figure 15](image-url)
6.2 Effect of Change in Uplink Transmit Power on UAV Communication Links of Various Antenna Techniques

As shown in Figure 16, the channel capacities of UAV uplink communication increases with increase in transmit power level of UAV. There is substantial difference between massive MIMO channel capacities at different uplink transmission power levels when compared to other multiple antenna techniques.

![Figure 16](image)

6.3 Effect of Change in Variance of True Channel in Massive MIMO Enabled UAV Communication Link

When the variance of true channel is increased from 10 dB to 40 dB, the performance of massive MIMO based UAV communication link also increases. The same is depicted in Figure 17.
6.4 Effect of increase in Number of Antennas on Massive MIMO enabled UAV Communication Link

Figure 18 shows that the performance of massive MIMO based UAV communication link increases exponentially with increase in number of base station antennas.
7. Conclusion

This paper has surveyed various aspects of provisioning of cellular communication support to UAVs. It has brought out the opportunities, challenges and inadequacies of integrating UAVs with existing cellular networks. By utilizing multiple antennas at ground base stations, these inadequacies of existing cellular networks can be mitigated. Utilization of multiple antennas in cellular networks offers wider coverage, higher data rates and security in wireless connectivity to UAVs. This paper brings out performance evaluation of UAV communication links using basic multiple antenna techniques and covers shortcomings of using point to point MIMO and MU-MIMO. However, this paper is particularly focused on massive MIMO based connectivity to UAVs. Utilization of hundreds of antennas at base station of massive MIMO based cellular networks, enhances the performance of communication links of UAV with base station. Enhanced performance of communication links generates plethora of opportunities for UAV operations and applications.

8. Compliance with Ethical Standards

FUNDING: No specific grant has been received for this research from any commercial, public or non-profit funding agencies.

CONFLICT OF INTEREST: It is declared by authors that there is no conflict of interest.

ETHICAL APPROVAL: No study with human participants executed by any of the authors is included in this article.

INFORMED CONSENT: Not applicable.

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Mozaffari, Saad, Bennis, and Debbah, “Mobile unmanned aerial vehicles (UAVs) for energy-efficient Internet of Things communications,” *IEEE Communication.,* vol. 16, pp. 7574–7589, 2017.


